Phase-Shift Characteristics of Helical Phase Shifters

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Abstract—**The phase shifters considered consist of a helix surrounded by or surrounding a ferrite toroid. The ferrites work at their maximum remanent magnetization. It is shown that the helix surrounding a ferrite allows at any combination of frequency and helix diameter a larger differential phase shift than the helix surrounded by a ferrite does; and that in the latter helix the phase shift is easily disturbed by the TE_{11} mode. It is furthermore shown that the $3\lambda/4$ per turn helix offers a larger fractional bandwidth than the $\lambda/4$ per turn helix does unless one uses ferrites with very low saturation magnetization. The theoretical results are supplemented by experimental data.**

Introduction

The HELICAL PHASE shifters dealt with here consist either of a helix surrounded by or surrounding a ferrite toroid, remanent at saturation magnetization. In both cases, differential phase shift can be obtained when the electrical length of the helix is $(2m+1)\lambda/4$ per turn. Suhl and Walker [1] have analyzed the propagation characteristics of a helix surrounded by a ferrite toroid. Hair [2] has studied and developed a phase shifter with a $3\lambda/4$ per turn helix surrounding a ferrite toroid.

These phase shifters are in general smaller and lighter than others, e.g., those using ferrite toroids in rectangular waveguides. The smallness is an advantage at frequencies below, a handicap at frequencies above, roughly 8 GHz. The helix surrounded by a ferrite toroid allows good thermal contact between the ferrite and the phase-shifter body. Thus it is relatively easy to keep the ferrite cool.

In the following, the propagation characteristics of a helix surrounded by and a helix surrounding a ferrite toroid are compared with each other; for the plane helix, the limiting case of both helices, the frequency dependence of $\lambda/4$ and $3\lambda/4$ per turn helical phase shifter are derived. The theoretical results are obtained with graphical methods and compared with experimental results.

Normal and Inverted Helices

A helix surrounding a ferrite toroid shall be called a "normal" helix, a helix surrounded by a ferrite toroid shall be called an "inverted" helix. The ferrite is magnetized circumferentially and—because of its tensor permeability—gives rise to different propagation constants $\beta_\pm$ (in the direction of the helix axis) for forward and backward traveling waves. The differential propagation constant or phase-shift coefficient $\Delta\beta=\beta_+-\beta_-$ determines the differential phase shift between forward and backward wave.

Manuscript received August 3, 1965; revised September 7, 1965.
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According to Fig. 1, the helix diameter (for infinitely thin wire) is r_0 for both helices, the helix pitch is p, and $\psi=\text{arc cot}\,(2\pi r_0/p)$ is the pitch angle. ϵ_n, ϵ_f, μ_n, $\boldsymbol{\mu}_f$ are the relative dielectric constants and permeabilities of the nonferrite medium and the ferrite. Generally $\mu_n=1$, while $\boldsymbol{\mu}_f$ is a tensor defined by

$$B=\mu_0\boldsymbol{\mu}_f H \text{ with } H=\begin{vmatrix} H_x \\ H_y \\ H_z \end{vmatrix} \text{ and}$$

$$\boldsymbol{\mu}_f=\begin{vmatrix} \mu & -jk & 0 \\ jk & \mu & 0 \\ 0 & 0 & 1 \end{vmatrix}. \tag{1}$$

The tensor elements are

$$\mu=1-\frac{\dfrac{\omega_m}{\omega}\dfrac{\omega_h}{\omega}}{1-\left(\dfrac{\omega_h}{\omega}\right)^2},\qquad k=\frac{\dfrac{\omega_m}{\omega}}{1-\left(\dfrac{\omega_h}{\omega}\right)^2} \tag{2}$$

where $\omega_h=\gamma H$ is proportional to the external dc magnetic field H, $\omega_m=\gamma M_s$ is proportional to the saturation magnetization of the ferrite, ω is the signal angular frequency, and γ is the gyromagnetic ratio. $\beta_0=\omega\sqrt{\mu_0\epsilon_0}$ is the propagation constant in free space.

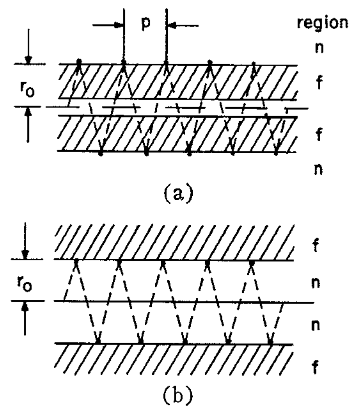

Fig. 1. Helix type phase shifters; (a) normal helix, (b) inverted helix.

Suhl, Walker and Hair have calculated the propagation constants for helices idealized as follows:

1) The helix and the ferrite toroid are infinitely long.
2) The helix is infinitely thin, directly on the ferrite surface and can be treated as a sheath helix.

3) Helix and ferrite and the whole device in general are lossless.
4) The propagation constant β for axial direction, the quantity to be found, is very large (slow wave) compared with all other propagation constants.
5) All fields are independent of the circumferential angle, i.e., only the zero-order mode of propagation on a helix is present.
6) The ferrite wall is so thick that in the normal helix the center hole can be neglected, and that in the inverted helix the wall can be assumed to be infinitely thick.

Furthermore, it is assumed by Hair and here:

7) The device works at remanent magnetization.
8) The remanent magnetization is equal to the saturation magnetization.
9) The gyromagnetic resonance frequency of the ferrite is lower than the signal frequency, i.e., the device works "below resonance."

The theory treats the switching wire and the shield surrounding the actual phase shifter as nonexistent. With these assumptions the determinantal equations for the propagation constants are for the normal helix as given by Hair

$$\left(\frac{\beta}{\beta_0 \cot \psi}\right)^2 = \frac{\epsilon_n \dfrac{K_1(\beta r_0)}{K_0(\beta r_0)} + \epsilon_f \dfrac{I_1(\beta r_0)}{I_0(\beta r_0)}}{\dfrac{1}{\mu_n} \dfrac{K_0(\beta r_0)}{K_1(\beta r_0)} + \dfrac{1}{\left(1 \pm \dfrac{\omega_m}{\omega}\right)} \dfrac{M_0(2\beta r_0)}{M_1(2\beta r_0)}} \tag{3}$$

and for the inverted helix as given by Suhl and Walker

$$\left(\frac{\beta}{\beta_0 \cot \psi}\right)^2 = \frac{\epsilon_n \dfrac{I_1(\beta r_0)}{I_0(\beta r_0)} + \epsilon_f \dfrac{K_1(\beta r_0)}{K_0(\beta r_0)}}{\dfrac{1}{\mu_n} \dfrac{I_0(\beta r_0)}{I_1(\beta r_0)} + \dfrac{1}{\left(1 \pm \dfrac{\omega_m}{\omega}\right)} \dfrac{W_0(2\beta r_0)}{W_1(2\beta r_0)}} \cdot \tag{4}$$

I_0 (and K_0), I_1 (and K_1) are zero and first-order modified Bessel functions, with finite values at $\beta r_0 = 0$ (or $\beta r_0 \rightarrow \infty$, respectively). M_0 (and W_0), M_1 (and W_1) are zero and first-order hypergeometric functions with finite values at $2\beta r_0 = 0$ (or $2\beta r_0 \rightarrow \infty$, respectively). The expressions become for the plane helix ($r_0 \rightarrow \infty$)

$$\left(\frac{\beta}{\beta_0 \cot \psi}\right)^2 = \frac{\epsilon_n + \epsilon_f}{\dfrac{1}{\mu_n} + \dfrac{1}{\left(1 \pm \dfrac{\omega_m}{\omega}\right)}} \cdot \tag{5}$$

In Fig. 2 the reduced propagation constants $\beta_{red} = \beta_\pm / \beta_0 \cot\psi$ are plotted vs. ω_m/ω with the reduced helix radius $r_{red} = r_0\beta_0 \cot\psi$ as parameter. Figure 2(a) for the normal helix is a copy of the graph given by Hair [2], while Fig. 2(b) for the inverted helix was obtained with $\epsilon_f = 12$ from Figs. 5 and 13 of Suhl's and Walker's report [1] for β as a function of the externally applied field H. From Fig. 2, the reduced phase-shift coefficient $\Delta\beta_{red} = \Delta\beta/\beta_0 \cot\psi = (\beta_+ - \beta_-)/\beta_0 \cot\psi$ is readily obtained and plotted in Fig. 3 vs. ω_m/ω with $r_0\beta_0 \cot \psi$ as parameters. The most interesting result is that the normal helix possesses a larger phase-shift coefficient than the inverted helix does for any combination of ω_m/ω and helix diameter. For $r_0\beta_0 \cot \psi \geq 5$, the plane helix is a good approximation for both the normal and the inverted helices.

It was assumed that β is very large compared with any other propagation constant. Figure 2 shows that for the backward traveling wave (β_-) the assumption is violated for $\omega_m/\omega > 0.7$. There the results are not reliable. Figure 3 shows that for $\omega_m/\omega < 0.6$ the reduced phase shift coefficient increases approximately linearly with ω_m/ω.

λ/4 and 3λ/4 Per Turn Helices

The reduced phase-shift coefficient of Fig. 2 has been derived by excluding all but the zero-order helix mode. Consequently $\Delta\beta_{red}$ allows a differential phase shift for all frequencies for which $\omega_m/\omega < 1$. In practice, however, frequencies with maximum and zero phase shift occur, due to the presence of higher-order helix modes. In particular, at $\lambda/4$ per turn the zero-order mode with positive phase velocity is predominant and gives rise to positive differential phase shift, while at $3\lambda/4$ per turn the first-order helix mode with negative phase velocity prevails, giving rise to negative differential phase shift. At $\lambda/2$ per turn both modes are equally strong and produce no phase shift. This effect can be accounted for by multiplying the $\Delta\beta(\omega)$ function with a polarization function

$$P(\omega) = \sin\left(\frac{\omega}{c}\sqrt{\epsilon_e}\, l_t\right) \text{ with} \tag{6}$$

$$l_t = 2\pi r_0 \sqrt{1 + \left(\frac{p}{2\pi r_0}\right)^2} \approx 2\pi r_0$$

where c is the velocity of light in free space, ϵ_e is the effective relative dielectric constant for the helix, and l_t is the helix length per turn. For a ferrite of length L the total differential phase shift is

$$\Delta\phi/\text{rad} = \Delta\beta P L \sqrt{\epsilon_e}. \tag{7}$$

The differential line length is obtained by multiplying the phase shift by $\lambda/2\pi$, where λ is the free space wavelength at angular frequency ω.

$$\Delta^l = (\Delta\phi/\text{rad})\, (\lambda/2\pi). \tag{8}$$

For comparisons it is convenient to normalize differential phase shift and line length as follows:

Reduced phase-shift coefficient

$$\Delta\beta_{red} = (\beta_+ - \beta_-)/\beta_0 \cot \psi \sim \omega_0/\omega. \tag{9}$$

Normalized phase-shift coefficient

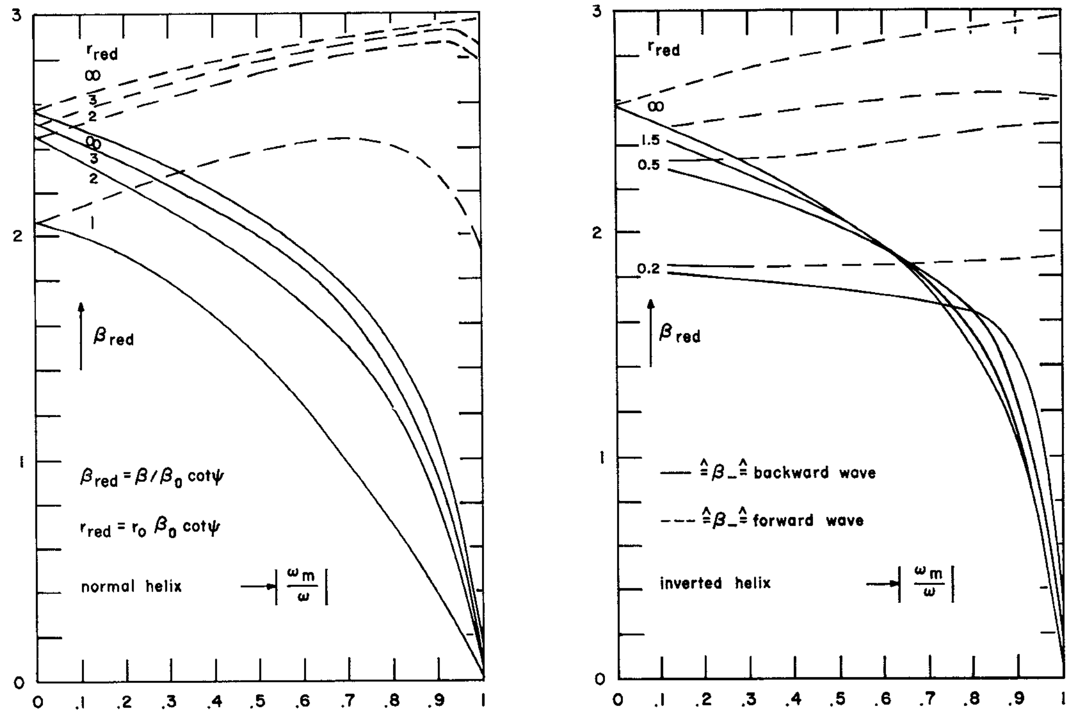

Fig. 2. Reduced propagation constants.

$$\Delta\beta_{\text{norm}} = \Delta\beta_{\text{red}}\,\omega/\omega_0 = \Delta\beta(\omega)/\beta_0(\omega_0)\cot\psi \sim \text{const.} \quad (10)$$

Normalized differential phase shift

$$\Delta\phi_{\text{norm}} = \Delta\beta_{\text{norm}}P(\omega) = \Delta\beta_{\text{norm}}\sin\left(n\frac{\pi}{2}\frac{\omega}{\omega_0}\right)$$

$$\sim \sin(\omega/\omega_0) \quad \text{for } n\lambda/4 \text{ per turn at } \omega_0. \quad (11)$$

Normalized differential line length

$$\Delta^{l}_{\text{norm}} = \Delta\phi_{\text{norm}}\lambda/\lambda_0 = \Delta\beta_{\text{red}}P(\omega) \sim \sin(\omega/\omega_0)/(\omega/\omega_0) \quad (12)$$

with λ_0 = free space wavelength at ω_0. The proportionality expressions are approximations for $\Delta\beta_{\text{red}} < 0.6$. For a plane helix these four quantities are plotted vs. ω/ω_0 in Fig. 4. According to Figs. 2 and 3 one should expect a large phase shift at the gyromagnetic resonance frequency $\omega = \omega_m$. As already noted, these curves do not hold true there. Besides, the ferrite losses can not be neglected there. Lax and Button ([3], Fig. 4-4) illustrate the susceptibility for circularly polarized fields in an unbounded ferrite medium. At the resonance frequency x_-', the loss causing part, goes through a maximum and x_-'', the phase-shift causing part, shows a zero. Corrections due to this gyromagnetic resonance are indicated in Fig. 4 in dotted lines.

The $3\lambda/4$ per turn helix shows a pronounced resonance effect for both the differential phase-shift and line length curves. The resonance frequency is nearly independent of the phase-shift coefficient. This means that the helix dimensions and the effective dielectric constant determine the center frequency. The $\lambda/4$ per turn helix shows a resonance effect for the phase shift only for $\omega_m/\omega_0 < 0.5$, where the phase shift per length is quite low. The differential line length shows no resonance effect. The differential phase-shift coefficient determines essentially the differential phase shift and line length function. The gyromagnetic resonance at $\omega = \omega_m$ is destructive. The most interesting result is that with a $\lambda/4$ per turn helix a broadband differential phase shift can be expected only for $\omega_m/\omega_0 < 0.5$ and that even then the differential line length is strongly frequency dependent.

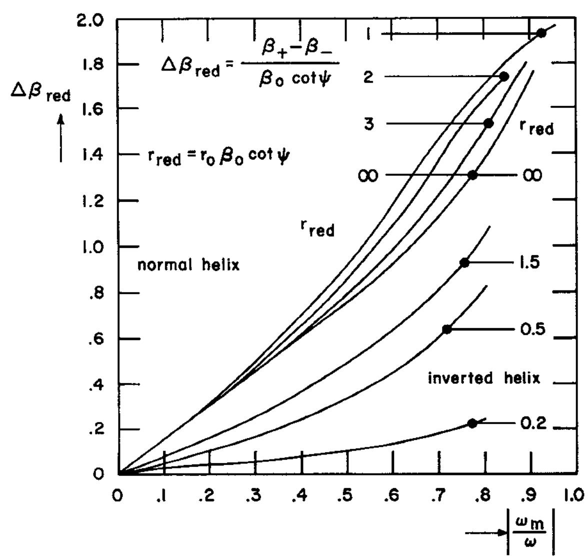

Fig. 3. Reduced differential phase-shift coefficient.

Experimental Results

Actual helix type phase shifters consist of the helix-ferrite combination and a conducting cylinder shielding this combination as indicated in Figs. 5 and 6. In addition to the helix mode, a TE_{11} mode can propagate between the helix and the shield. For normal helices the

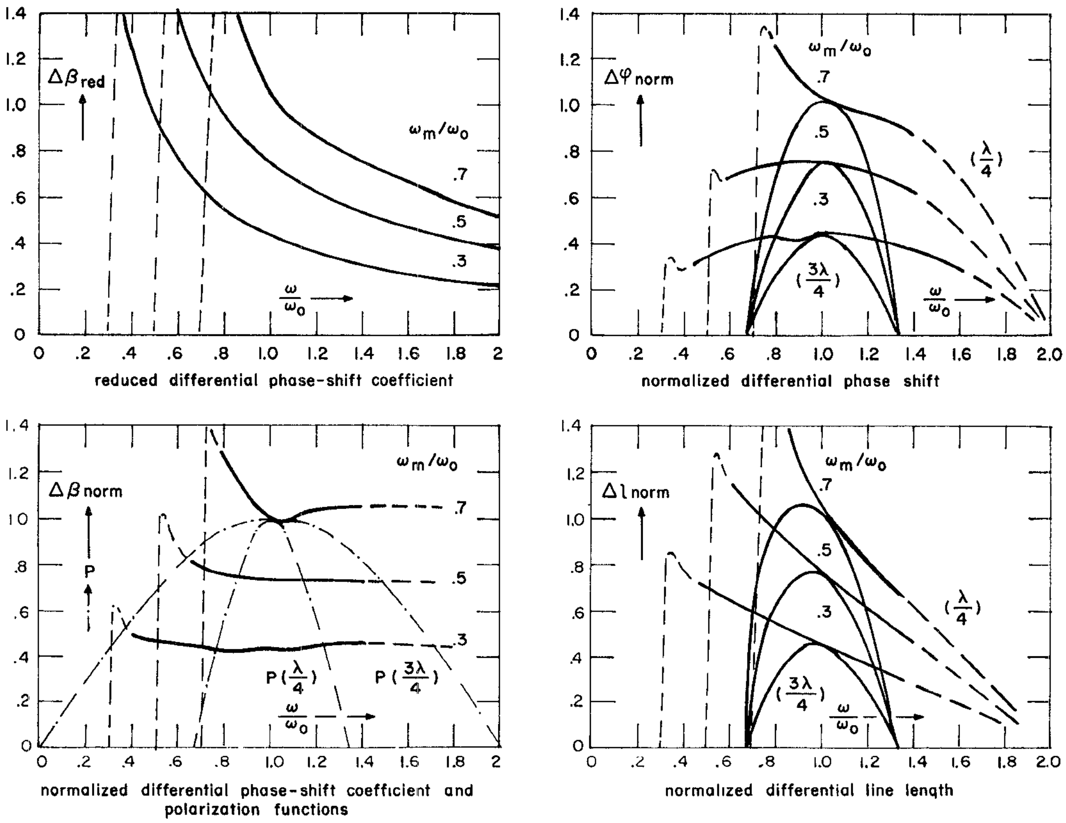

Fig. 4. Phase-shift characteristics for plane helix type phase shifters.

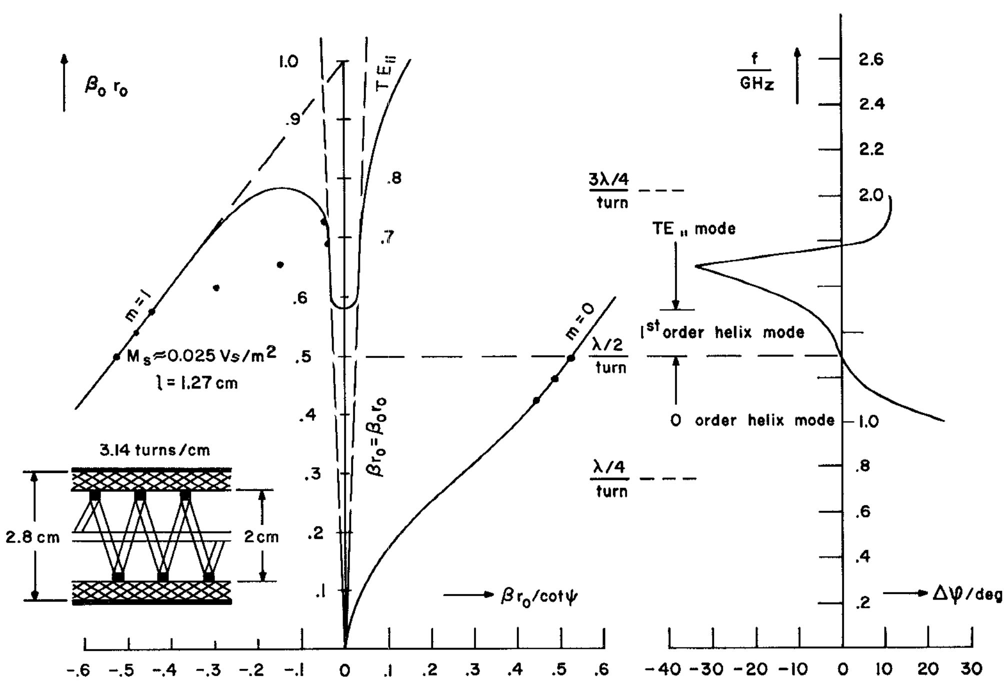

Fig. 5. Mode chart and differential phase-shift characteristic for an inverted helix type phase shifter.

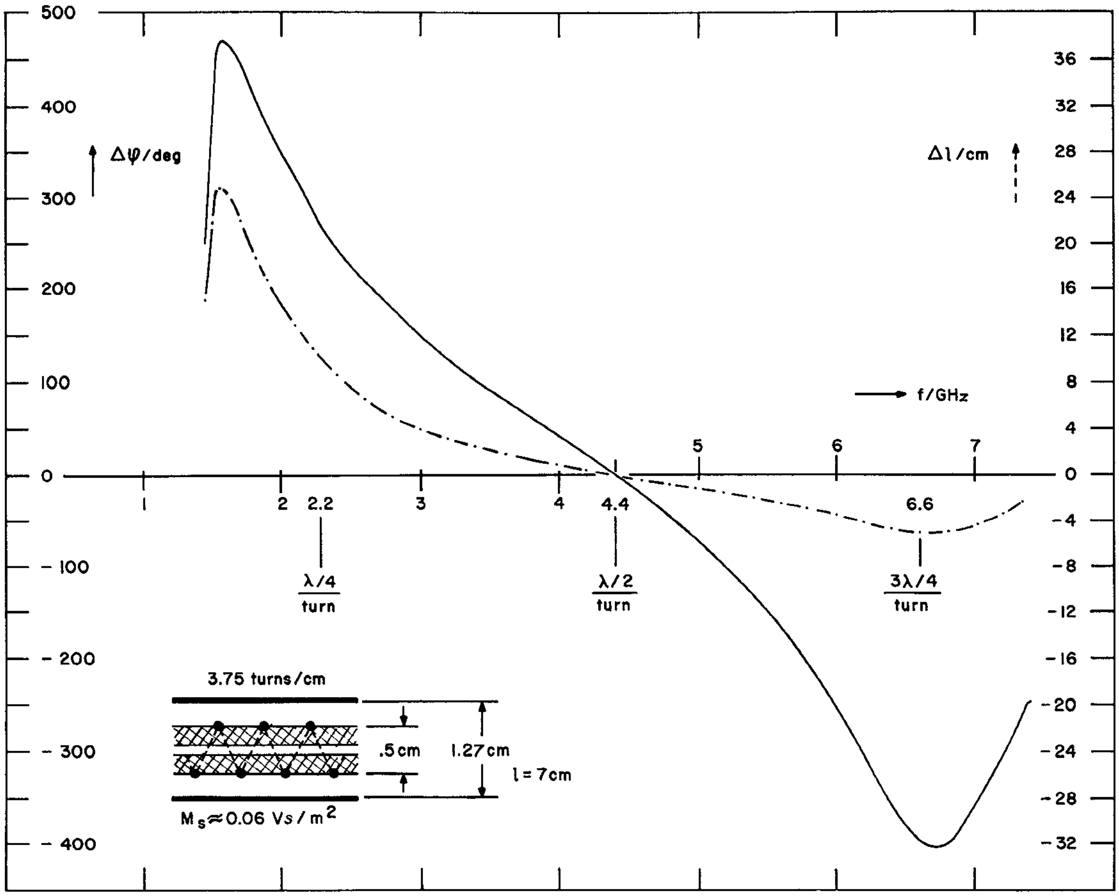

Fig. 6. Differential phase shift and line length characteristics for a normal helix type phase shifter.

cutoff condition for the TE_{11} mode (the mean circumference is approximately one wavelength in the unbounded medium) is usually reached at frequencies higher than those for which the helix modes experience $3\lambda/4$ per turn. In the inverted helix the TE_{11} mode can propagate at lower frequencies and is therefore damaging. Figure 5 shows the mode chart [4] and a typical differential phase shift vs. frequency characteristic for an inverted helix. A good agreement between measured insertion length and propagation characteristic has been achieved only at frequencies below the TE_{11} cutoff frequency (about 1.3–1.4 GHz). Below 1.3 GHz the zero-order helical mode prevails, at 1.3 GHz the first-order mode becomes predominant, but at only slightly higher frequencies coupling to the fast TE_{11} mode begins and the phase shift decays. The $3\lambda/4$ per turn resonance is suppressed by the TE_{11} mode. The device is an extremely narrow-band one.

Figure 6 shows the differential line length and phase shift for a normal helix. The results correspond to the theory. The larger bandwidth is obtained at $3\lambda/4$ per turn and the TE_{11} mode takes over at about 7.4 GHz, i.e., above the $3\lambda/4$ per turn range.

Conclusions

"Normal" helix type phase shifters are superior to "inverted" helix type ones because they offer a higher differential phase shift per length and they do not suffer as easily from TE_{11} modes. The poor electrical characteristics of the inverted helix type phase shifter offset the advantage of having a good thermal contact between ferrite and phase-shifter body in this device. In the $3\lambda/4$ per turn helix the phase shift vs. frequency behavior is essentially determined by the device geometry and effective dielectric constant. In the $\lambda/4$ per turn helix the phase-shift coefficient determines mainly the frequency dependence. The gyromagnetic resonance effect is destructive, unless $\omega_m/\omega_0 < 0.5$. Under this same condition the $3\lambda/4$ per turn helix offers the larger fractional bandwidth.

Acknowledgment

It is a pleasure to thank Dr. H. C. Rothenberg and H. A. Hair for having encouraged this investigation.

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